Lista 1

Exercício 1:

$$\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{a} \to (\mathbf{b} \to \mathbf{c}))^{(1)} \quad \mathbf{a}^{(3)}}{(\mathbf{b} \to \mathbf{c})}(E\to) \quad \mathbf{b}^{(2)}}{\mathbf{c}}(E\to)}{(\mathbf{a} \to \mathbf{c})}(I\to)(3)}{(\mathbf{b} \to (\mathbf{a} \to \mathbf{c}))}(I\to)(2)}{((\mathbf{a} \to (\mathbf{b} \to \mathbf{c})) \to (\mathbf{b} \to (\mathbf{a} \to \mathbf{c})))}(I\to)(1)$$

Exercício 2:

$$\cfrac{\cfrac{\cfrac{\mathbf{a}^{(1)} \quad \cfrac{\mathbf{a}^{(1)}}{(\mathbf{a} \vee \mathbf{b})}(I\vee)}{(\mathbf{a} \wedge (\mathbf{a} \vee \mathbf{b}))}(I\wedge)}{(\mathbf{a} \to (\mathbf{a} \wedge (\mathbf{a} \vee \mathbf{b})))}(I\to)(1) \qquad \cfrac{\cfrac{(\mathbf{a} \wedge (\mathbf{a} \vee \mathbf{b}))^{(2)}}{\mathbf{a}}(E\wedge)}{((\mathbf{a} \wedge (\mathbf{a} \vee \mathbf{b})) \to \mathbf{a})}(I\to)(2)}{((\mathbf{a} \wedge (\mathbf{a} \vee \mathbf{b})) \leftrightarrow \mathbf{a})}(I\wedge)$$

Exercício 3:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{c} \to \mathbf{a})^{(1)} \quad \mathbf{c}^{(3)}}{\mathbf{a}}(E\to) \quad \cfrac{(\mathbf{c} \to \mathbf{b})^{(2)} \quad \mathbf{c}^{(3)}}{\mathbf{b}}(E\to)}{(\mathbf{a} \wedge \mathbf{b})}(I\wedge)}{(\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b}))}(I\to)(3)}{((\mathbf{c} \to \mathbf{b}) \to (\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b})))}(I\to)(2)}{((\mathbf{c} \to \mathbf{a}) \to ((\mathbf{c} \to \mathbf{b}) \to (\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b}))))}(I\to)(1)$$

Exercício 4:

$$\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{a} \wedge \mathbf{b})^{(2)}}{\mathbf{a}}(E\wedge)}{((\mathbf{a} \wedge \mathbf{b}) \to \mathbf{a})}(I\to)(2) \qquad \cfrac{\cfrac{\mathbf{a}^{(3)} \quad \cfrac{(\mathbf{a} \to \mathbf{b})^{(1)} \quad \mathbf{a}^{(3)}}{\mathbf{b}}(E\to)}{(\mathbf{a} \wedge \mathbf{b})}(I\wedge)}{(\mathbf{a} \to (\mathbf{a} \wedge \mathbf{b}))}(I\to)(3)}{\cfrac{((\mathbf{a} \wedge \mathbf{b}) \leftrightarrow \mathbf{a})}{((\mathbf{a} \to \mathbf{b}) \to ((\mathbf{a} \wedge \mathbf{b}) \leftrightarrow \mathbf{a}))}(I\to)(1)}(I\wedge) \qquad \cfrac{\cfrac{\cfrac{\cfrac{((\mathbf{a} \wedge \mathbf{b}) \leftrightarrow \mathbf{a})^{(4)}}{(\mathbf{a} \to (\mathbf{a} \wedge \mathbf{b}))}(E\wedge) \quad \mathbf{a}^{(5)}}{(\mathbf{a} \wedge \mathbf{b})}(E\to)}{\mathbf{b}}(E\wedge)}{(\mathbf{a} \to \mathbf{b})}(I\to)(5)}{(((\mathbf{a} \wedge \mathbf{b}) \leftrightarrow \mathbf{a}) \to (\mathbf{a} \to \mathbf{b}))}(I\to)(4)}{((\mathbf{a} \to \mathbf{b}) \leftrightarrow ((\mathbf{a} \wedge \mathbf{b}) \leftrightarrow \mathbf{a}))}(I\wedge)$$

Exercício 5:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{a} \to (\mathbf{b} \to \mathbf{c}))^{(1)} \quad \mathbf{a}^{(3)}}{(\mathbf{b} \to \mathbf{c})}(E\to) \quad \cfrac{(\mathbf{a} \to \mathbf{b})^{(2)} \quad \mathbf{a}^{(3)}}{\mathbf{b}}(E\to)}{\mathbf{c}}(E\to)}{(\mathbf{a} \to \mathbf{c})}(I\to)(3)}{((\mathbf{a} \to \mathbf{b}) \to (\mathbf{a} \to \mathbf{c}))}(I\to)(2)}{((\mathbf{a} \to (\mathbf{b} \to \mathbf{c})) \to ((\mathbf{a} \to \mathbf{b}) \to (\mathbf{a} \to \mathbf{c})))}(I\to)(1)$$

Exercício 6:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b}))^{(1)} \quad \mathbf{c}^{(2)}}{(\mathbf{a} \wedge \mathbf{b})}(E\to)}{\mathbf{a}}(E\wedge)}{(\mathbf{c} \to \mathbf{a})}(I\to)(2) \quad \cfrac{\cfrac{\cfrac{(\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b}))^{(1)} \quad \mathbf{c}^{(3)}}{(\mathbf{a} \wedge \mathbf{b})}(E\to)}{\mathbf{b}}(E\wedge)}{(\mathbf{c} \to \mathbf{b})}(I\to)(3)}{((\mathbf{c} \to \mathbf{a}) \wedge (\mathbf{c} \to \mathbf{b}))}(I\wedge)}{((\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b})) \to ((\mathbf{c} \to \mathbf{a}) \wedge (\mathbf{c} \to \mathbf{b})))}(I\to)(1) \qquad \cfrac{\cfrac{\cfrac{\cfrac{\cfrac{((\mathbf{c} \to \mathbf{a}) \wedge (\mathbf{c} \to \mathbf{b}))^{(4)}}{(\mathbf{c} \to \mathbf{a})}(E\wedge) \quad \mathbf{c}^{(5)}}{\mathbf{a}}(E\to) \quad \cfrac{\cfrac{((\mathbf{c} \to \mathbf{a}) \wedge (\mathbf{c} \to \mathbf{b}))^{(4)}}{(\mathbf{c} \to \mathbf{b})}(E\wedge) \quad \mathbf{c}^{(5)}}{\mathbf{b}}(E\to)}{(\mathbf{a} \wedge \mathbf{b})}(I\wedge)}{(\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b}))}(I\to)(5)}{(((\mathbf{c} \to \mathbf{a}) \wedge (\mathbf{c} \to \mathbf{b})) \to (\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b})))}(I\to)(4)}{((\mathbf{c} \to (\mathbf{a} \wedge \mathbf{b})) \leftrightarrow ((\mathbf{c} \to \mathbf{a}) \wedge (\mathbf{c} \to \mathbf{b})))}(I\wedge)$$

Lista 2

Exercício 1:

$$\cfrac{\cfrac{\cfrac{(\mathbf{a}\vee\mathbf{b})^{(2)}\quad\cfrac{\cfrac{(\mathbf{a}\to\mathbf{b})^{(1)}\quad\mathbf{a}^{(3)}}{\mathbf{b}}(E\to)\quad\mathbf{b}^{(4)}}{\mathbf{b}}(E\vee)(3)(4)}{\cfrac{((\mathbf{a}\vee\mathbf{b})\to\mathbf{b})}{((\mathbf{a}\vee\mathbf{b})\leftrightarrow\mathbf{b})}}(I\to)(2)\quad\cfrac{\cfrac{\mathbf{b}^{(5)}}{(\mathbf{a}\vee\mathbf{b})}(I\vee)}{(\mathbf{b}\to(\mathbf{a}\vee\mathbf{b}))}(I\to)(5)}{(I\wedge)}}{((\mathbf{a}\to\mathbf{b})\to((\mathbf{a}\vee\mathbf{b})\leftrightarrow\mathbf{b}))}(I\to)(1)\quad\cfrac{\cfrac{\cfrac{\cfrac{((\mathbf{a}\vee\mathbf{b})\leftrightarrow\mathbf{b})^{(6)}}{((\mathbf{a}\vee\mathbf{b})\to\mathbf{b})}(E\wedge)\quad\cfrac{\mathbf{a}^{(7)}}{(\mathbf{a}\vee\mathbf{b})}(I\vee)}{\mathbf{b}}(E\to)}{(\mathbf{a}\to\mathbf{b})}(I\to)(7)}{(((\mathbf{a}\vee\mathbf{b})\leftrightarrow\mathbf{b})\to(\mathbf{a}\to\mathbf{b}))}(I\to)(6)}{((\mathbf{a}\to\mathbf{b})\leftrightarrow((\mathbf{a}\vee\mathbf{b})\leftrightarrow\mathbf{b}))}(I\wedge)$$

Exercício 2:

$$\cfrac{\cfrac{(\mathbf{a}\vee(\mathbf{a}\wedge\mathbf{b}))^{(1)}\quad\mathbf{a}^{(2)}\quad\cfrac{(\mathbf{a}\wedge\mathbf{b})^{(3)}}{\mathbf{a}}(E\wedge)}{\mathbf{a}}(E\vee)(2)(3)}{((\mathbf{a}\vee(\mathbf{a}\wedge\mathbf{b}))\to\mathbf{a})}(I\to)(1)$$

Exercício 3:

$$\cfrac{\cfrac{\cfrac{(\mathbf{a}\wedge(\mathbf{b}\vee\mathbf{c}))^{(1)}}{(\mathbf{b}\vee\mathbf{c})}(E\wedge)\quad\cfrac{\mathbf{b}^{(2)}\quad\cfrac{(\mathbf{a}\wedge(\mathbf{b}\vee\mathbf{c}))^{(1)}}{\mathbf{a}}(E\wedge)}{\cfrac{(\mathbf{a}\wedge\mathbf{b})}{((\mathbf{a}\wedge\mathbf{b})\vee(\mathbf{a}\wedge\mathbf{c}))}(I\vee)}(I\wedge)\quad\cfrac{\cfrac{\cfrac{(\mathbf{a}\wedge(\mathbf{b}\vee\mathbf{c}))^{(1)}}{\mathbf{a}}(E\wedge)\quad\mathbf{c}^{(3)}}{(\mathbf{a}\wedge\mathbf{c})}(I\wedge)}{((\mathbf{a}\wedge\mathbf{b})\vee(\mathbf{a}\wedge\mathbf{c}))}(I\vee)}{((\mathbf{a}\wedge\mathbf{b})\vee(\mathbf{a}\wedge\mathbf{c}))}(E\vee)(2)(3)}{((\mathbf{a}\wedge(\mathbf{b}\vee\mathbf{c}))\to((\mathbf{a}\wedge\mathbf{b})\vee(\mathbf{a}\wedge\mathbf{c})))}(I\to)(1)$$

Exercício 4:

$$\cfrac{\cfrac{\cfrac{(\mathbf{a}\vee(\mathbf{b}\wedge\mathbf{c}))^{(1)}\quad\cfrac{\mathbf{a}^{(2)}}{(\mathbf{a}\vee\mathbf{c})}(I\vee)\quad\cfrac{\cfrac{(\mathbf{b}\wedge\mathbf{c})^{(3)}}{\mathbf{c}}(E\wedge)}{(\mathbf{a}\vee\mathbf{c})}(I\vee)}{(\mathbf{a}\vee\mathbf{c})}(E\vee)(2)(3)\quad\cfrac{(\mathbf{a}\vee(\mathbf{b}\wedge\mathbf{c}))^{(1)}\quad\cfrac{\mathbf{a}^{(4)}}{(\mathbf{a}\vee\mathbf{b})}(I\vee)\quad\cfrac{\cfrac{(\mathbf{b}\wedge\mathbf{c})^{(5)}}{\mathbf{b}}(E\wedge)}{(\mathbf{a}\vee\mathbf{b})}(I\vee)}{(\mathbf{a}\vee\mathbf{b})}(E\vee)(4)(5)}{((\mathbf{a}\vee\mathbf{b})\wedge(\mathbf{a}\vee\mathbf{c}))}(I\wedge)}{((\mathbf{a}\vee(\mathbf{b}\wedge\mathbf{c}))\to((\mathbf{a}\vee\mathbf{b})\wedge(\mathbf{a}\vee\mathbf{c})))}(I\to)(1)$$

Exercício 5:

$$\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{a}\vee\mathbf{b})^{(3)}\quad\cfrac{(\mathbf{a}\to\mathbf{c})^{(1)}\quad\mathbf{a}^{(4)}}{\mathbf{c}}(E\to)\quad\cfrac{(\mathbf{b}\to\mathbf{c})^{(2)}\quad\mathbf{b}^{(5)}}{\mathbf{c}}(E\to)}{\mathbf{c}}(E\vee)(4)(5)}{((\mathbf{a}\vee\mathbf{b})\to\mathbf{c})}(I\to)(3)}{((\mathbf{b}\to\mathbf{c})\to((\mathbf{a}\vee\mathbf{b})\to\mathbf{c}))}(I\to)(2)}{((\mathbf{a}\to\mathbf{c})\to((\mathbf{b}\to\mathbf{c})\to((\mathbf{a}\vee\mathbf{b})\to\mathbf{c})))}(I\to)(1)$$

Exercício 6:

$$\cfrac{\cfrac{\cfrac{(\mathbf{a}\vee\mathbf{b})^{(2)}\quad\cfrac{\cfrac{((\mathbf{a}\to\mathbf{c})\wedge(\mathbf{b}\to\mathbf{c}))^{(1)}}{(\mathbf{a}\to\mathbf{c})}(E\wedge)\quad\mathbf{a}^{(3)}}{\mathbf{c}}(E\to)\quad\cfrac{\cfrac{((\mathbf{a}\to\mathbf{c})\wedge(\mathbf{b}\to\mathbf{c}))^{(1)}}{(\mathbf{b}\to\mathbf{c})}(E\wedge)\quad\mathbf{b}^{(4)}}{\mathbf{c}}(E\to)}{\mathbf{c}}(E\vee)(3)(4)}{((\mathbf{a}\vee\mathbf{b})\to\mathbf{c})}(I\to)(2)}{(((\mathbf{a}\to\mathbf{c})\wedge(\mathbf{b}\to\mathbf{c}))\to((\mathbf{a}\vee\mathbf{b})\to\mathbf{c}))}(I\to)(1)$$

Lista 3

Exercício 1:

$$\cfrac{\cfrac{\cfrac{\cfrac{\mathbf{a}^{(1)} \quad \neg\mathbf{a}^{(2)}}{\bot}(I\bot)}{\neg\neg\mathbf{a}}(I\neg)(2)}{(\mathbf{a} \to \neg\neg\mathbf{a})}(I\to)(1) \quad \cfrac{\cfrac{\neg\neg\mathbf{a}^{(3)}}{\mathbf{a}}(E\neg)}{(\neg\neg\mathbf{a} \to \mathbf{a})}(I\to)(3)}{(\mathbf{a} \leftrightarrow \neg\neg\mathbf{a})}(I\wedge)$$

Exercício 2:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{a} \to \mathbf{b})^{(1)} \quad \mathbf{a}^{(3)}}{\mathbf{b}}(E\to) \quad \neg\mathbf{b}^{(2)}}{\bot}(I\bot)}{\neg\mathbf{a}}(I\neg)(3)}{(\neg\mathbf{b} \to \neg\mathbf{a})}(I\to)(2)}{((\mathbf{a} \to \mathbf{b}) \to (\neg\mathbf{b} \to \neg\mathbf{a}))}(I\to)(1)$$

Exercício 3:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\mathbf{a}^{(2)}}{(\mathbf{a} \vee \neg\mathbf{a})}(I\vee) \quad \neg(\mathbf{a} \vee \neg\mathbf{a})^{(1)}}{\bot}(I\bot)}{\neg\mathbf{a}}(I\neg)(2)}{(\mathbf{a} \vee \neg\mathbf{a})}(I\vee) \quad \neg(\mathbf{a} \vee \neg\mathbf{a})^{(1)}}{\bot}(I\bot)}{\neg\neg(\mathbf{a} \vee \neg\mathbf{a})}(I\neg)(1)}{(\mathbf{a} \vee \neg\mathbf{a})}(E\neg)$$

Exercício 4:

$$\cfrac{\cfrac{\cfrac{\mathbf{a}}{((\mathbf{a} \to \mathbf{b}) \to \mathbf{a})}(I\to)(2) \quad (\mathbf{a} \to \mathbf{b})}{\mathbf{a}}(E\to)}{(((\mathbf{a} \to \mathbf{b}) \to \mathbf{a}) \to \mathbf{a})}(I\to)(1)$$

Exercício 5:

$$\cfrac{\cfrac{\cfrac{\cfrac{(\neg\mathbf{a} \vee \neg\mathbf{b})^{(2)} \quad \cfrac{\cfrac{(\mathbf{a} \wedge \mathbf{b})^{(1)}}{\mathbf{a}}(E\wedge) \quad \neg\mathbf{a}^{(3)}}{\bot}(I\bot) \quad \cfrac{\cfrac{(\mathbf{a} \wedge \mathbf{b})^{(1)}}{\mathbf{b}}(E\wedge) \quad \neg\mathbf{b}^{(4)}}{\bot}(I\bot)}{\bot}(E\vee)(3)(4)}{\neg(\neg\mathbf{a} \vee \neg\mathbf{b})}(I\neg)(2)}{((\mathbf{a} \wedge \mathbf{b}) \to \neg(\neg\mathbf{a} \vee \neg\mathbf{b}))}(I\to)(1) \quad \cfrac{\cfrac{\cfrac{\cfrac{\neg\mathbf{a}^{(6)}}{(\neg\mathbf{a} \vee \neg\mathbf{b})}(I\vee) \quad \neg(\neg\mathbf{a} \vee \neg\mathbf{b})^{(5)}}{\bot}(I\bot)}{\neg\neg\mathbf{a}}(I\neg)(6)}{\mathbf{a}}(E\neg) \quad \cfrac{\cfrac{\cfrac{\neg\mathbf{b}^{(7)}}{(\neg\mathbf{a} \vee \neg\mathbf{b})}(I\vee) \quad \neg(\neg\mathbf{a} \vee \neg\mathbf{b})^{(5)}}{\bot}(I\bot)}{\neg\neg\mathbf{b}}(I\neg)(7)}{\mathbf{b}}(E\neg)}{\cfrac{(\mathbf{a} \wedge \mathbf{b})}{(\neg(\neg\mathbf{a} \vee \neg\mathbf{b}) \to (\mathbf{a} \wedge \mathbf{b}))}(I\to)(5)}(I\wedge)}{((\mathbf{a} \wedge \mathbf{b}) \leftrightarrow \neg(\neg\mathbf{a} \vee \neg\mathbf{b}))}(I\wedge)$$

Exercício 6:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{a} \to \mathbf{b})^{(1)} \quad \cfrac{(\mathbf{a} \wedge \neg\mathbf{b})^{(2)}}{\mathbf{a}}(E\wedge)}{\mathbf{b}}(E\to) \quad \cfrac{(\mathbf{a} \wedge \neg\mathbf{b})^{(2)}}{\neg\mathbf{b}}(E\wedge)}{\bot}(I\bot)}{\neg(\mathbf{a} \wedge \neg\mathbf{b})}(I\neg)(2)}{((\mathbf{a} \to \mathbf{b}) \to \neg(\mathbf{a} \wedge \neg\mathbf{b}))}(I\to)(1) \quad \cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\neg\mathbf{b}^{(5)} \quad \mathbf{a}^{(4)}}{(\mathbf{a} \wedge \neg\mathbf{b})}(I\wedge) \quad \neg(\mathbf{a} \wedge \neg\mathbf{b})^{(3)}}{\bot}(I\bot)}{\neg\neg\mathbf{b}}(I\neg)(5)}{\mathbf{b}}(E\neg)}{(\mathbf{a} \to \mathbf{b})}(I\to)(4)}{(\neg(\mathbf{a} \wedge \neg\mathbf{b}) \to (\mathbf{a} \to \mathbf{b}))}(I\to)(3)}{((\mathbf{a} \to \mathbf{b}) \leftrightarrow \neg(\mathbf{a} \wedge \neg\mathbf{b}))}(I\wedge)$$

Exercício 7:

$$\cfrac{\cfrac{(\neg\mathbf{a}\vee\neg\mathbf{b})}{(\neg(\mathbf{a}\wedge\mathbf{b})\to(\neg\mathbf{a}\vee\neg\mathbf{b}))}(I\!\to\!)(1)\qquad \cfrac{(\neg\mathbf{a}\vee\neg\mathbf{b})^{(2)}\qquad \cfrac{\cfrac{(\mathbf{a}\wedge\mathbf{b})^{(3)}}{\mathbf{a}}(E\wedge)\quad \neg\mathbf{a}^{(4)}}{\bot}(I\bot)\quad \cfrac{\cfrac{(\mathbf{a}\wedge\mathbf{b})^{(3)}}{\mathbf{b}}(E\wedge)\quad \neg\mathbf{b}^{(5)}}{\bot}(I\bot)}{\cfrac{\cfrac{\bot}{\neg(\mathbf{a}\wedge\mathbf{b})}(I\neg)(3)}{((\neg\mathbf{a}\vee\neg\mathbf{b})\to\neg(\mathbf{a}\wedge\mathbf{b}))}(I\!\to\!)(2)}(E\vee)(4)(5)}{(\neg(\mathbf{a}\wedge\mathbf{b})\leftrightarrow(\neg\mathbf{a}\vee\neg\mathbf{b}))}(I\wedge)$$

Exercício 8:

$$\cfrac{\cfrac{(\mathbf{a}\vee\mathbf{b})^{(1)}\quad \cfrac{\mathbf{a}^{(3)}\quad \cfrac{(\neg\mathbf{a}\wedge\neg\mathbf{b})^{(2)}}{\neg\mathbf{a}}(E\wedge)}{\bot}(I\bot)\quad \cfrac{\mathbf{b}^{(4)}\quad \cfrac{(\neg\mathbf{a}\wedge\neg\mathbf{b})^{(2)}}{\neg\mathbf{b}}(E\wedge)}{\bot}(I\bot)}{\cfrac{\cfrac{\bot}{\neg(\neg\mathbf{a}\wedge\neg\mathbf{b})}(I\neg)(2)}{((\mathbf{a}\vee\mathbf{b})\to\neg(\neg\mathbf{a}\wedge\neg\mathbf{b}))}(I\!\to\!)(1)}(E\vee)(3)(4)\qquad \cfrac{(\mathbf{a}\vee\mathbf{b})}{(\neg(\neg\mathbf{a}\wedge\neg\mathbf{b})\to(\mathbf{a}\vee\mathbf{b}))}(I\!\to\!)(5)}{((\mathbf{a}\vee\mathbf{b})\leftrightarrow\neg(\neg\mathbf{a}\wedge\neg\mathbf{b}))}(I\wedge)$$

Exercício 9:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\mathbf{a}^{(2)}}{(\mathbf{a}\vee\mathbf{b})}(I\vee)\quad \neg(\mathbf{a}\vee\mathbf{b})^{(1)}}{\bot}(I\bot)}{\neg\mathbf{a}}(I\neg)(2)\quad \cfrac{\cfrac{\cfrac{\mathbf{b}^{(3)}}{(\mathbf{a}\vee\mathbf{b})}(I\vee)\quad \neg(\mathbf{a}\vee\mathbf{b})^{(1)}}{\bot}(I\bot)}{\neg\mathbf{b}}(I\neg)(3)}{\cfrac{(\neg\mathbf{a}\wedge\neg\mathbf{b})}{(\neg(\mathbf{a}\vee\mathbf{b})\to(\neg\mathbf{a}\wedge\neg\mathbf{b}))}(I\!\to\!)(1)}(I\wedge)\qquad \cfrac{(\mathbf{a}\vee\mathbf{b})^{(5)}\quad \cfrac{\mathbf{a}^{(6)}\quad \cfrac{(\neg\mathbf{a}\wedge\neg\mathbf{b})^{(4)}}{\neg\mathbf{a}}(E\wedge)}{\bot}(I\bot)\quad \cfrac{\mathbf{b}^{(7)}\quad \cfrac{(\neg\mathbf{a}\wedge\neg\mathbf{b})^{(4)}}{\neg\mathbf{b}}(E\wedge)}{\bot}(I\bot)}{\cfrac{\cfrac{\bot}{\neg(\mathbf{a}\vee\mathbf{b})}(I\neg)(5)}{((\neg\mathbf{a}\wedge\neg\mathbf{b})\to\neg(\mathbf{a}\vee\mathbf{b}))}(I\!\to\!)(4)}(E\vee)(6)(7)}{(\neg(\mathbf{a}\vee\mathbf{b})\leftrightarrow(\neg\mathbf{a}\wedge\neg\mathbf{b}))}(I\wedge)$$

Exercício 10:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(\mathbf{a}\to\mathbf{b})^{(1)}\quad \mathbf{a}^{(3)}}{\mathbf{b}}(E\!\to\!)}{(\neg\mathbf{a}\vee\mathbf{b})}(I\vee)\quad \neg(\neg\mathbf{a}\vee\mathbf{b})^{(2)}}{\bot}(I\bot)}{\neg\mathbf{a}}(I\neg)(3)}{(\neg\mathbf{a}\vee\mathbf{b})}(I\vee)\quad \neg(\neg\mathbf{a}\vee\mathbf{b})^{(2)}}{\bot}(I\bot)}{\cfrac{\cfrac{\neg\neg(\neg\mathbf{a}\vee\mathbf{b})}{(\neg\mathbf{a}\vee\mathbf{b})}(E\neg)}{((\mathbf{a}\to\mathbf{b})\to(\neg\mathbf{a}\vee\mathbf{b}))}(I\!\to\!)(1)}(I\neg)(2)}{}\qquad \cfrac{(\neg\mathbf{a}\vee\mathbf{b})^{(4)}\quad \cfrac{\cfrac{\mathbf{a}^{(5)}\quad \neg\mathbf{a}^{(6)}}{\bot}(I\bot)}{\mathbf{b}}(E\bot)\quad \mathbf{b}^{(7)}}{\cfrac{\cfrac{\mathbf{b}}{(\mathbf{a}\to\mathbf{b})}(I\!\to\!)(5)}{((\neg\mathbf{a}\vee\mathbf{b})\to(\mathbf{a}\to\mathbf{b}))}(I\!\to\!)(4)}(E\vee)(6)(7)}{((\mathbf{a}\to\mathbf{b})\leftrightarrow(\neg\mathbf{a}\vee\mathbf{b}))}(I\wedge)$$

$\boxed{\text{Lista 4}}$

Exercício 1:

$$\cfrac{\cfrac{\cfrac{\cfrac{\forall x.\mathbf{R}(x,x)^{(1)}}{\mathbf{R}(x,x)}(E\forall)}{\exists y.\mathbf{R}(x,y)}(I\exists)}{\forall x.\exists y.\mathbf{R}(x,y)}(I\forall)}{(\forall x.\mathbf{R}(x,x) \to \forall x.\exists y.\mathbf{R}(x,y))}(I\to)(1)$$

Exercício 2:

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\exists x.\forall y.\mathbf{R}(x,y)^{(1)} \quad \forall y.\mathbf{R}(x,y)^{(2)}}{\forall y.\mathbf{R}(x,y)}(E\exists)(2)}{\mathbf{R}(x,y)}(E\forall)}{\exists x.\mathbf{R}(x,y)}(I\exists)}{\forall y.\exists x.\mathbf{R}(x,y)}(I\forall)}{(\exists x.\forall y.\mathbf{R}(x,y) \to \forall y.\exists x.\mathbf{R}(x,y))}(I\to)(1)$$

Exercício 3: $\forall x.\mathbf{R}(x,x) \vdash \forall x.\exists y.\mathbf{R}(x,y)$

$$\cfrac{\cfrac{\cfrac{\cfrac{\forall x.\mathbf{R}(x,x)}{\mathbf{R}(x,x)}(E\forall)}{\exists y.\mathbf{R}(x,y)}(I\exists)}{\forall x.\exists y.\mathbf{R}(x,y)}(I\forall)}{}$$

Exercício 4: $\forall x.\mathbf{R}(x,x) \vdash \forall x.\forall y.\,(\mathbf{R}(x,y) \to \exists z.\,(\mathbf{R}(x,z) \land \mathbf{R}(z,y)))$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\mathbf{R}(x,y)^{(1)}}{\forall y.\mathbf{R}(x,y)}(I\forall)}{\forall x.\forall y.\mathbf{R}(x,y)}(I\forall)}{\forall y.\mathbf{R}(z,y)}(E\forall)}{\mathbf{R}(z,y)}(E\forall) \quad \cfrac{\cfrac{\cfrac{\mathbf{R}(x,y)^{(1)}}{\forall y.\mathbf{R}(x,y)}(I\forall)}{\mathbf{R}(x,z)}(E\forall)}{}}{(\mathbf{R}(x,z) \land \mathbf{R}(z,y))}(I\land)}{\exists z.\,(\mathbf{R}(x,z) \land \mathbf{R}(z,y))}(I\exists)}{(\mathbf{R}(x,y) \to \exists z.\,(\mathbf{R}(x,z) \land \mathbf{R}(z,y)))}(I\to)(1)}{\forall y.\,(\mathbf{R}(x,y) \to \exists z.\,(\mathbf{R}(x,z) \land \mathbf{R}(z,y)))}(I\forall)}{\forall x.\forall y.\,(\mathbf{R}(x,y) \to \exists z.\,(\mathbf{R}(x,z) \land \mathbf{R}(z,y)))}(I\forall)$$

Exercício 5: $\forall x.\forall y.\,(\mathbf{R}(x,y) \to \neg\mathbf{R}(y,x)) \vdash \forall x.\neg\mathbf{R}(x,x)$

$$\cfrac{\cfrac{\cfrac{\mathbf{R}(x,x)^{(1)} \quad \cfrac{\cfrac{\cfrac{\forall x.\forall y.\,(\mathbf{R}(x,y) \to \neg\mathbf{R}(y,x))}{\forall y.\,(\mathbf{R}(x,y) \to \neg\mathbf{R}(y,x))}(E\forall)}{(\mathbf{R}(x,x) \to \neg\mathbf{R}(x,x))}(E\forall) \quad \mathbf{R}(x,x)^{(1)}}{\neg\mathbf{R}(x,x)}(E\to)}{\bot}(I\bot)}{\neg\mathbf{R}(x,x)}(I\neg)(1)}{\forall x.\neg\mathbf{R}(x,x)}(I\forall)$$

Exercício 6: $\exists x.\mathbf{p}(x) \vdash \neg\forall x.\neg\mathbf{p}(x)$

$$\cfrac{\cfrac{\cfrac{\exists x.\mathbf{p}(x) \quad \mathbf{p}(x)^{(2)}}{\mathbf{p}(x)}(E\exists)(2) \quad \cfrac{\forall x.\neg\mathbf{p}(x)^{(1)}}{\neg\mathbf{p}(x)}(E\forall)}{\bot}(I\bot)}{\neg\forall x.\neg\mathbf{p}(x)}(I\neg)(1)$$

Exercício 7: $\exists x.\mathbf{p}(x,x) \vdash \exists x.\exists y.\mathbf{p}(x,y)$

5

$$\cfrac{\cfrac{\cfrac{\exists x.\mathbf{p}(x,x) \quad \mathbf{p}(x,x)^{(2)}}{\mathbf{p}(x,x)}(E\exists)(2)}{\cfrac{\exists y.\mathbf{p}(x,y)}{\mathbf{p}(x,y)}(I\exists) \quad \mathbf{p}(x,y)^{(1)}}{\cfrac{\mathbf{p}(x,y)}{\cfrac{\exists y.\mathbf{p}(x,y)}{\exists x.\exists y.\mathbf{p}(x,y)}(I\exists)}(I\exists)}(E\exists)(1)}$$

$\boxed{\text{Lista 5}}$

Exercício 1: $\forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(y,z)) \to \mathbf{R}(x,z)), \forall x.\neg\mathbf{R}(x,x) \vdash \forall x.\forall y. (\mathbf{R}(x,y) \to \neg\mathbf{R}(y,x))$

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{
\cfrac{
\cfrac{\forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(y,z)) \to \mathbf{R}(x,z))}{\cfrac{\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(y,z)) \to \mathbf{R}(x,z))}{\cfrac{\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(y,z)) \to \mathbf{R}(x,z))}{((\mathbf{R}(x,y) \wedge \mathbf{R}(y,x)) \to \mathbf{R}(x,x))}(E\forall)}(E\forall)}(E\forall) \qquad \cfrac{\mathbf{R}(x,y)^{(1)} \quad \mathbf{R}(y,x)^{(2)}}{(\mathbf{R}(x,y) \wedge \mathbf{R}(y,x))}(I\wedge)}{\mathbf{R}(x,x)}(E\to) \qquad \cfrac{\forall x.\neg\mathbf{R}(x,x)}{\neg\mathbf{R}(x,x)}(E\forall)}{\bot}(I\bot)}{\neg\mathbf{R}(y,x)}(I\neg)(2)}{(\mathbf{R}(x,y) \to \neg\mathbf{R}(y,x))}(I\to)(1)}{\forall y. (\mathbf{R}(x,y) \to \neg\mathbf{R}(y,x))}(I\forall)}{\forall x.\forall y. (\mathbf{R}(x,y) \to \neg\mathbf{R}(y,x))}(I\forall)
$$

Exercício 2: $\forall x.\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x)) \vdash \forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(x,z)) \to \exists u. (\mathbf{R}(y,u) \wedge \mathbf{R}(z,u)))$

$$
\cfrac{
\cfrac{
\cfrac{
\cfrac{
\cfrac{
\cfrac{\cfrac{\forall x.\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x))}{\cfrac{\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x))}{(\mathbf{R}(x,z) \to \mathbf{R}(z,x))}(E\forall)}(E\forall) \quad \cfrac{(\mathbf{R}(x,y) \wedge \mathbf{R}(x,z))^{(1)}}{\mathbf{R}(x,z)}(E\wedge)}{\mathbf{R}(z,x)}(E\to) \qquad \cfrac{\cfrac{\forall x.\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x))}{\cfrac{\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x))}{(\mathbf{R}(x,y) \to \mathbf{R}(y,x))}(E\forall)}(E\forall) \quad \cfrac{(\mathbf{R}(x,y) \wedge \mathbf{R}(x,z))^{(1)}}{\mathbf{R}(x,y)}(E\wedge)}{\mathbf{R}(y,x)}(E\to)}{(\mathbf{R}(y,x) \wedge \mathbf{R}(z,x))}(I\wedge)}{\exists u. (\mathbf{R}(y,u) \wedge \mathbf{R}(z,u))}(I\exists)}{((\mathbf{R}(x,y) \wedge \mathbf{R}(x,z)) \to \exists u. (\mathbf{R}(y,u) \wedge \mathbf{R}(z,u)))}(I\to)(1)}{\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(x,z)) \to \exists u. (\mathbf{R}(y,u) \wedge \mathbf{R}(z,u)))}(I\forall)}{\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(x,z)) \to \exists u. (\mathbf{R}(y,u) \wedge \mathbf{R}(z,u)))}(I\forall)}{\forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(x,z)) \to \exists u. (\mathbf{R}(y,u) \wedge \mathbf{R}(z,u)))}(I\forall)
$$

Exercício 3: $\forall x.\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x)), \forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(y,z)) \to \mathbf{R}(x,z)) \vdash \forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(x,z)) \to \mathbf{R}(y,z))$

$$
\text{(árvore de dedução natural do Exercício 3)}
$$

Exercício 4: $\forall x.\exists y.\mathbf{R}(x,y), \forall x.\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x)), \forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(y,z)) \to \mathbf{R}(x,z)) \vdash \forall x.\mathbf{R}(x,x)$

$$
\text{(árvore de dedução natural do Exercício 4)}
$$

Exercício 5: $\forall x.\forall y.\forall z. ((\mathbf{R}(x,y) \wedge \mathbf{R}(x,z)) \to \mathbf{R}(y,z)), \forall x.\mathbf{R}(x,x) \vdash \forall x.\forall y. (\mathbf{R}(x,y) \to \mathbf{R}(y,x))$

$$\cfrac{\cfrac{\cfrac{\cfrac{\forall x.\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)\right)}{\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)\right)}(E\forall)}{\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)\right)}(E\forall)}{(\mathbf{R}(x,y)\wedge\mathbf{R}(x,x))\to\mathbf{R}(y,x)}(E\forall) \qquad \cfrac{\cfrac{\forall x.\mathbf{R}(x,x)}{\mathbf{R}(x,x)}(E\forall) \quad \mathbf{R}(x,y)^{(1)}}{(\mathbf{R}(x,y)\wedge\mathbf{R}(x,x))}(I\wedge)}{\mathbf{R}(y,x)}(E\to)$$

$$\cfrac{\cfrac{\cfrac{\cfrac{\mathbf{R}(y,x)}{(\mathbf{R}(x,y)\to\mathbf{R}(y,x))}(I\to)(1)}{\forall y.\left(\mathbf{R}(x,y)\to\mathbf{R}(y,x)\right)}(I\forall)}{\forall x.\forall y.\left(\mathbf{R}(x,y)\to\mathbf{R}(y,x)\right)}(I\forall)}{}$$

Exercício 6: $\forall x.\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)\right),\forall x.\mathbf{R}(x,x)\vdash\forall x.\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(y,z))\to\mathbf{R}(x,z)\right)$

$$\cfrac{\cfrac{\cfrac{\forall x.\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)\right)}{\cfrac{\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)\right)}{\cfrac{\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)\right)}{(\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))\to\mathbf{R}(y,z)}(E\forall)}(E\forall)}}{}(E\forall) \qquad \cfrac{\cfrac{(\mathbf{R}(x,y)\wedge\mathbf{R}(y,z))^{(1)}}{\mathbf{R}(x,y)}(E\wedge) \qquad \cfrac{\cfrac{\cfrac{(\mathbf{R}(x,y)\wedge\mathbf{R}(y,z))^{(1)}}{\mathbf{R}(y,z)}(E\wedge)}{\forall y.\mathbf{R}(y,z)}(I\forall)}{\mathbf{R}(x,z)}(E\forall)}{(\mathbf{R}(x,y)\wedge\mathbf{R}(x,z))}(I\wedge)}{\mathbf{R}(y,z)}(E\to)$$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\mathbf{R}(y,z)}{\forall y.\mathbf{R}(y,z)}(I\forall)}{\mathbf{R}(x,z)}(E\forall)}{((\mathbf{R}(x,y)\wedge\mathbf{R}(y,z))\to\mathbf{R}(x,z))}(I\to)(1)}{\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(y,z))\to\mathbf{R}(x,z)\right)}(I\forall)}{\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(y,z))\to\mathbf{R}(x,z)\right)}(I\forall)}{\forall x.\forall y.\forall z.\left((\mathbf{R}(x,y)\wedge\mathbf{R}(y,z))\to\mathbf{R}(x,z)\right)}(I\forall)$$